Algebraic Topology Qual

May 21, 2018

Problem 1. Suppose X is a path-connected space with universal covering space $\tilde{X}$. Prove that if $\tilde{X}$ is compact then $\pi_1(X)$ is finite.

Problem 2. Find a Δ-complex structure for the Klein bottle and compute its simplicial homology with coefficients in $\mathbb{Z}$.

Problem 3.

- What is $H_i(S^3; \mathbb{Q})$ for $i \geq 0$? Just the answer; no justification necessary.
- A closed 3-manifold M is called a *rational homology 3-sphere* if $H_i(M; \mathbb{Q}) = H_i(S^3; \mathbb{Q})$ for all i. Prove (using a combination of Poincaré duality and the Universal Coefficient Theorem) that a closed 3-manifold M is a rational homology 3-sphere iff $H_1(M; \mathbb{Q}) = 0$.

Problem 4. Let $X = S^1 \vee S^1 \vee S^1$ be the wedge of three circles shown below. Let x, y, z be the three loops indicated in the figure. Let $W = X \cup_{f_1} e_1^2 \cup_{f_2} e_2^2$ be the space obtained from X by attaching one 2-cell via the map

$$f_1 : \partial e_1^2 \to X$$

which sends the boundary to the loop $xyx^{-1}zy^{-1}$; and attaching another 2-cell via the map

$$f_2 : \partial e_2^2 \to X$$

which sends the boundary to the loop z^7.

- Describe the associated cellular chain complex for W (including the boundary maps).
- Compute $H^i(W; \mathbb{Z}/2\mathbb{Z})$ for all $i \geq 0$.

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Please show all your work. You may use any results proved in class or on HW.

(1) Let X be the vector field $X = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}$ on $\mathbb{R}^2$ and let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function $f(x, y) = x^2 + y^2$.

(a) Compute $df(X)$ in terms of the standard coordinates x, y on $\mathbb{R}^2$. (Please begin by computing df.)

(b) Compute