Leaving No Ethical Value Behind: Triage Protocol Design for Pandemic Rationing

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Abstract

Rationing of medical resources is a critical issue in the COVID-19 pandemic. Most existing triage protocols are based on a priority point system, in which a formula specifies the order in which the supply of a resource, such as a ventilator, is to be rationed for patients. A priority point system generates an identical priority ranking specifying claims on all units. Triage protocols in some states (e.g. Michi50e.gy ystatesfroicalstateshealthstateswcaltr((e.givJ/F22 1.151 Td [(T)8

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certain patients are excluded, there should be a priority order for patients based on estimated mortality risk and such patient's priority status should be re-evaluated every 48 hours based on changes in health status (Zucker et al., 2015). Mortality risk is measured by the Sequential Organ Failure Assessment (SOFA) score[3]. At any priority level, there is the potential that one priority group could completely exhaust the remaining available resources. In cases of excess demand upon remaining resources by a given priority group, New York and other proposals recommended random allocation { a lottery { among equal-priority patients (Zucker et al., 2015; Emanuel et al., 2020).

Several criticize priority point systems solely based on SOFA for ignoring multiple ethical values. Task forces commissioned to develop guidelines recognize the need to integrate a variety ethical values and advocate for a multi-principle approach, see, e.g., White et al. (2009) and Daugherty-Biddison et al. (2017). For example, a Johns Hopkins study examining public perceptions of di erent ethical principles summarized (Biddison et al., 2013):

> Both groups suggested alternative strategies, such as organ transplantation allocation criteria as a model or adopting a tiered approach by applying di erent principles at di erent stages in process.

To integrate multiple ethical values, White et al. (2009) formulates a multi-principle priority point system. Using a similar aggregation methodology as was used to construct the SOFA score, in this mechanism several ethical values are put on a numeric scale and summed up across ethical values to arrive aae asc0(et(re-ev)r009#-400y1gus)-416(prioa)-38(wius)-32equenusedet an 18(e(a)-32p priorities can be the same. This system balances their interests against other ethical goals.

Reserve systems are widespread in resource allocation settings outside of medicine when there are conflicting objectives.[5] The key idea of a reserve system is to divide the total supply into several categories, and consider allocation for these smaller number of units separately. Specific objectives can be realized within these categories, using explicit priorities or randomization. The advantage of a reserve system is in its flexibility. A priority point system obscures trade-offs between different principles because it aggregates several different considerations into a single priority score. It is even possible that one principle might dominate other principles unexpectedly depending on how scores are scaled.

It is important to note that we are agnostic on what the reserve types or sizes should be. Our primary aim is to inform the debate on how a reserve system can be used to balance competing objectives, and provide a route forward in several high-stakes debates on rationing. We do, however, discuss some reserve categories below only to indicate possibilities, and leave the final decisions to medical ethicists, task forces, and other stakeholders.

We first illustrate the power of a reserve system by explaining how it can assist with the debate on whether frontline health workers should obtain priority for vital medical resources. Under Michigan's guidelines, essential personnel are prioritized for these resources (Michigan, 2012). Ethicists have also emphasize its importance in the current pandemic (see, e.g., Emanuel prohibited from receiving vital resources, it violates a fundamental principle of non-exclusion. That is, it violates the idea that every patient, no matter his or her circumstances, should have some hope of obtaining a life-saving resource. In a reserve system, if a portion of vital resources

illustrate how it works, consider a hypothetical patient with a SOFA score of seven. She obtains two points based on the ethical value of saving the most lives. If the patient has no chronic diseases or comorbidities and is between 61-74 years old, she obtains four more points based on the other two ethical values yielding a total of six. A patient with a lower total point score has a higher priority for the resource than a patient with a higher total point score.[11] Between the SOFA based priority point system and White et al. (2009) multi-principle point system, more than half of US states use a priority point system (Whyte, 2020).

The strength of the priority point mechanism is simplicity. Each ethical value is represented with a monotonic integer valued function. Values are then integrated with an additive formula producing an aggregate point score for each patient. The claims of patients over medical resources are determined based on their point scores, with a lower score typically indicating a higher claim. While practical, priority point mechanisms are limiting for a number of reasons.

First, priority points norm or scale di erent and potentially-unrelated ethical principles into one dimensioon

depending on the choice of the mechanism, and not all of these mechanisms have an intuitive interpretation. This multiplicity resulted in the emergence of a subclass of these mechanisms in real-life applications of these problem, where categories are processed sequentially for a given order of categories. In the context of medical resource rationing, our focus is this intuitive subclass of reserve systems we call *sequential reserve matching rules*. As a result, for our main application of interest, there is one additional parameter of a reserve system: the processing sequence of categories. This parameter plays an important role in the distribution of the units, and so we elaborate on its relevance next.

2.3 Reserve Category Processing Sequence

Sequential reserve matching rules were first formally introduced by Kominers and Sönmez (2016) in a more general environment with heterogenous units and multiple terms of allocation. Although not life-and-death situations, reserve systems are widespread in real-life applications including the implementation of affirmative action policies in school choice in Boston (Dur et al., 2018), Chicago (Dur, Pathak and Sönmez, 2019), the implementation of reservation policies in India (Sönmez and Yenmez, 2019a,b), and the allocation of immigration visas in the U.S. (Pathak, Rees-Jones and Sönmez, 2020). As shown in these studies, the processing order of reserve categories is a key parameter with significant distributional implications.[13]

To explain intuitively why processing order is important, imagine a simple scenario in which there are 60 ventilators. A medical ethics committee decides that there are two important principles: equal treatment of equals and prioritizing essential medical personnel. Based on their view, they define a reserve category for essential medical personnel, which reserves 50% of ventilators for them. Within this reservation, there is random allocation via lottery. The remaining 50% of ventilators are unreserved and open to all patients, including essential personnel. These are also allocated via lottery. Suppose that there are 60 essential personnel who need a ventilator and 60 other patients who do as well. If the essential personnel reserve category is processed patients. The 30 remaining ventilators are all reserved and allocated to essential personnel. This results assigning 45 ventilators to essential personnel and 15 ventilators to other patients. Thus in this simple example, the choice of the processing sequence of categories is a matter of life or death for 5 essential medical personnel and 5 members of the general community.

As this simple example illustrates, our application to triage protocol design is another setting where reserve processing matters. Indeed, understanding the implications of reserve category processing order is especially critical in our application given the emphasis on transparency. Much of our theoretical analysis in Section 3 relates to this subtle aspect of sequential reserve matching rules. Perhaps the most important lesson from this analysis is that the later a reserve category is processed the better it is for its bene ciaries. This important feature, also apparent in the example above, has the following important implications for design. If a reserve category is intended as a \boost" for a group of participants, then the category should be processed after more inclusive categories open to all. This form of implementing reserve policies can be interpreted as an over-and-above policy In contrast, if a reserve category is intended as a \protective measure" for a group of participants, then the category should be processed after more inclusive categories open to all. This second form of implementing reserve policies can be interpreted as a minimum guarantee policy.[14]

2.4 Potential Reserve Categories

The parameters of a reserve system can be modi ed for di erent medical resources. Emanuel et al. (2020) emphasizes that \prioritization guidelines should di er by intervention and should respond to changing scienti c evidence.

principles.

2.4.1 Essential Personnel Category

The essential personnel category provides some form of priority to personnel such as frontline health workers. Essential personnel may have made potentially life-saving contributions to society in the past, and they are presently subject to severe risks. Therefore, ethicists justify this reserve on the basis of both reciprocity and instrumental value. Furthermore, Emanuel et al. (2020) o ers the following incentive-based rationale for prioritizing essential personnel:

> : : : but giving them priority for ventilators recognizes their assumption of the high-risk work of saving others, and it may also discourage absenteeism.

Nevertheless, essential personnel are not prioritized in several state guidelines. One of the main justi cations for denying essential personnel priority is articulated in 2015 New York State Ventilator Allocation Guidelines (Zucker et al., 2015):

> Expanding the category of privilege to include all the workers listed above may mean that only health care workers obtain access to ventilators in certain communities. This approach may leave no ventilators for community members, including children; this alternative was unacceptable to the Task Force.

Limiting priority allocation of ventilators to essential personnel for only a subset of ventilators is a natural compromise, compared to the two extreme policies that either provide it for all units (e.g. Michigan) or for none of the units (e.g. New York State and Minnesota).

2.4.2 Good Samaritan Reciprocity Category

Another possible category is aGood Samaritan reciprocity category, which provides priority based on Good Samaritan acts. In such a category, a small fraction of resources are reserved for those who have saved lives through their past Good Samaritan acts. These could be participants for clinical trials on vaccine or treatment development (Emanuel et al., 2020), altruistic donors who have donated their kidneys to a stranger, or people who have donated large quantities of blood over the years. Good samaritan status can also be provided for compatible patient-donor pairs who voluntarily participate in kidney exchange even though they do not have to, and save another patient's life who was incompatible with his/her donor. This type of incentive could save a large number of lives. Sönmez, Ünver and Yenmez (2020) estimate a 180 additional kidney patients could receive living donor transplants for every 10 percent of compatible pairs who can be incentivized to participate in kidney exchange. A state task force can determine which acts \deserve" a Good Samaritan status.

In addition to the widely-accepted ethical principle of reciprocity, this category can also be motivated by the incentives it creates. If the aim is to maximize this incentive, it could be

2.4.3 Protective Reserve Categories: Disabled and Disadvantaged

Disabilities rights advocates have opposed rationing plans based on expected health outcomes using survival probabilities because such criteria are inherently discriminatory.[15] Persad (2020) recounts that several prefer either random selection or minimal triage that completely ignores any differences in likelihood or magnitude of benefit, or the likely quantity of resources required for benefit. A reserve system allows for a resolution of this dispute. In particular, a disabled protective category can be established for disabled patients reserving some of the units for these groups. If the representatives of these groups reach a decision to implement random lottery within disabled patients for these units, this can be implemented under a reserve system without interfering with the priority order for units in other categories.

Another criticism of priority point systems which use mortality risk or comorbidities as part of the priority score is that these criteria do not take into account differences in expected health outcomes driven by discrimination in access to health care or other social inequalities. For instance, disparate access to testing for disadvantaged groups may increase COVID-19 prevalence in these communities (Blow, 2020). A reserve system can be used to accommodate this perspective. A portion of scarce resources could be set aside in the form of a disadvantaged protective category

> tions should be used as the first tiebreaker, with priority going to younger patients. We recommend the following categories: age 12-40, age 41-60, age 61-75, older than age 75. We also recommend that individuals who are vital to the acute care response be given priority, which could be operationalized in the form of a tiebreaker.

The Pittsburgh system illustrates that preferential treatment for essential personnel can be

These are the patients in I^0 who receive units under matching.

In real-life applications of our model, it is important to allocate units to qualified individuals without wasting any, and following the priorities attached to these units. We next formulate this idea through three axioms:

Definition 1 A matching $\in \mathcal{M}$ is *individually rational* if, for any $i \in I$ and $c \in C$,

$$(i) = c \implies i \quad _c ; :$$

Our first axiom formulates the idea that individuals should only receive those units for which

Observe that in our hypothetical market, all the primitives introduced so far naturally follows from the primitives of the original problem. The only primitive of the hypothetical market that is somewhat "artificial" is the next one:

Each patient $i \in I$ has a strict preference relation $\succ_i$ over the set $C \cup \{\emptyset\}$, such that, for each patient $i \in I$,

$$c \succ_i \emptyset \iff \text{patient } i \text{ is eligible for category } c.$$

While in the original problem a patient is indifferent between all units (and therefore all categories as well), in the hypothetical market she has strict preferences between the categories. This "flexibility" in the construction of the hypothetical market is the basis of our main characterization.

For each patient $i \in I$, let $\mathcal{P}_i$ be the set of all preferences constructed in this way, and let $\mathcal{P} = \times_{i \in I} \mathcal{P}_i$.

Given a preference profile $\succ = (\succ_i)_{i \in I}$, the individual-proposing deferred-acceptance algorithm (DA) produces a matching as follows.

Individual Proposing Deferred Acceptance Algorithm (DA)

Step 1: Each patient in I applies to her most preferred category among categories for which she is eligible. Suppose that I_c^1 is the set of patients who apply to category c. Category c tentatively assigns applicants with the highest priority according to π_c until all patients in I_c^1 are chosen or all r_c units are allocated, whichever comes first, and permanently rejects the rest. If there are no rejections, then stop.

Step k: Each patient who was rejected in Step k-1 applies to her next preferred category among categories for which she is eligible, if such a category exists. Suppose that I_c^k is the union of the set of patients who were tentatively assigned to category c in Step k-1 and the set of patients who just proposed to category c. Category c tentatively assigns patients in I_c^k with the highest priority according to π_c until all patients in I_c^k are chosen or all r_c units are allocated, whichever comes first, and permanently rejects the rest. If there are no rejections, then stop.

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3.2 Sequential Reserve Matching

An interpretation of the DA-induced matchings is helpful to motivate in focusing a subset of these matchings. Recall that the hypothetical two-sided matching market constructed above relies on an artificial preference profile $(\succ_i)_{i \in I}$ of patients over categories. What this corresponds to under the DA algorithm is that any patient i is considered for categories that deem her eligible in sequence, following the ranking of these categories under her artificial preferences $\succ_i$. Unless there is a systematic way to construct these preferences, it may be difficult to motivate adopting this methodology for real-life applications. For example, if a patient i is considered first for an unreserved category and then for an essential personnel category, whereas another patient j with similar characteristics is considered for them in the reverse order, it may be difficult to justify this practice. That is, while there are a potentialg Sequen

Given an order of precedence $\triangleleft \in \Delta$, the induced sequential reserve matching is individually rational, non-wasteful, and it respect priorities. Thus, it is DA-induced by Theorem 1. Indeed it corresponds to a very specific DA-induced matching.

Proposition 1 *Fix an order of precedence $\triangleleft \in \Delta$. Let the preference profile $\succ^{\triangleleft} \in \mathcal{P}$ be such that, for each patient $i \in I$ and pair of categories $c, c' \in C$,*

$$c \succ_i^{\triangleleft} c' \iff c \triangleleft c'.$$

Then the sequential reserve matching $\varphi_{\triangleleft}$ is DA-induced from the preference profile $\succ^{\triangleleft}$.

3.3 Comparative Statics for Sequential Reserve Matching

In many real-life applications such as affirmative action in school choice and H1-B visa allocation, there is a baseline priority order π of individuals. This priority order may depend on scores on a standardized exam, a random lottery, or arrival time of application. In our main application of pandemic resource allocation, it may depend on SOFA scores described in Section 2.1. This baseline priority order is used to construct the priority order for each of the reserve categories, although each category except one gives preferential treatment to a specific subset of individuals. For example, in our main application these could be essential personnel, Good Samaritans, or people with disabilities. In this section, we focus on this subclass of rationing problems and present an analysis of sequential reserve matching on this class.

To formulate this subclass, we designate a **beneficiary group** $I_c \subseteq I$ for each category $c \in C$. It is assumed that all patients in its beneficiary group are eligible for a category. That is, for any $c \in C$ and $i \in I_c$,

$$i \; \pi_c \; \emptyset.$$

There is a category $u \in C$, called the **unreserved** category, which has all patients as its beneficiaries and endowed with the same priority order as the baseline priority order. That is,

$$I_u = I \quad \text{and} \quad \pi_u = \pi.$$

Any other category $c \in C \setminus \{u\}$, referred to as a **preferential treatment** category, has a more restrictive set $I_c \subsetneq I$ of beneficiaries and it is endowed with a priority order π_c with the following structure: for any pair of patients $i, i' \in I$,

$$i \in I_c \text{ and } i' \in I \setminus I_c \implies i \; \pi_c \; i',$$
$$i, i' \in I_c \text{ and } i \; \pi \; i' \implies i \; \pi_c \; i', \quad \text{and}$$
$$i, i' \in I \setminus I_c \text{ and } i \; \pi \; i' \implies i \; \pi_c \; i'.$$

Function $\tau : I \rightarrow \mathcal{C} \setminus \{u\} \cup \{\emptyset\}$ identifies which preferential category each patient is a beneficiary of (if any). Here, for any patient $i \in I$,

- $\tau(i) = c$ for some $c \in \mathcal{C} \setminus \{u\}$ means that patient i is a beneficiary of the preferential treatment category c and the unreserved category u, whereas

- $\tau(i) = \emptyset$ means that patient i is only a beneficiary of the unreserved category u.

Let I_g, referred to as the set of *general-community patients*, be the set of patients who are each a beneficiary of the unreserved category only:

$$I_g = \{i \in I : \tau(i) = \emptyset\} = I \setminus \cup_{c \in \mathcal{C} \setminus \{u\}} I_c.$$

We refer to these problems as *rationing problems induced by the baseline priority order* π.

In particular two types are such problems have widespread applications.

We say that a priority profile $(\pi_c)_{c \in \mathcal{C}}$ has *soft reserves* if, for any category $c \in \mathcal{C}$ and any patient $i \in I$,

$$i \; \pi_c \; \emptyset.$$

Under a soft-reserves rationing problem all individuals are eligible for all categories. This is the case, for example, in our main application of pandemic resource allocation.

We say that a priority profile $(\pi_c)_{c \in \mathcal{C}}$ has *hard reserves* if, for any preferential treatment category $c \in \mathcal{C} \setminus \{u\}$,

1. $i \; \pi_c \; \emptyset$ for any of its beneficiaries $i \in I_c$, whereas

2. $\emptyset \; \pi_c \; i$ for any patient $i \in I \setminus I_c$ who is not a beneficiary.

Under a hard-reserves rationing problem, only the beneficiaries of a preferred treatment category are eligible for units in this category. This is the case, for example, in H1-B visa allocation in the US.

Allocation rules based on sequential reserve matching are used in a range of practical applications. However, it is important to pay attention to the choice of the order of precedence in these problems, for it has potentially significant distributional implications. [18]

We obtain the sharpest results on the choice of order of precedence for the case of hard reserves. Therefore in the remainder of this section, we focus on this case. However, since our main application of pandemic rationing is one with soft reserves, we present a version of Theorem 2 in Theorem 5 of Appendix A. Although Theorem 5 is a theoretically weaker result, it is equally relevant for our main application of pandemic rationing. In the same Appendix, we also present two counterexamples showing that the stronger version of the result fails to hold once the hard-reserves assumption is dropped.

Our next result shows that the later a preferential treatment category is processed, the more favorable it is for its beneficiaries at the expense of everyone else.

[18] See, for instance, the example in Section 2.3 for an illustration.

Theorem 2 Assuming each patient is a beneficiary of at most one preferential treatment category, consider a hard-reserves rationing problem induced by a baseline priority order. Fix a preferential treatment category $c \in \mathcal{C} \setminus \{u\}$, another category $c' \in \mathcal{C} \setminus \{c\}$, and a pair of orders of precedence $\triangleright, \triangleright' \in \Pi$ such that:

- $c' \triangleright c$,
- $c \triangleright' c'$, and
- for any $\hat{c} \in \mathcal{C}$ and $\tilde{c} \in \mathcal{C} \setminus \{c, c'\}$,

$$\hat{c} \triangleright \tilde{c} \iff \hat{c} \triangleright' \tilde{c}.$$

That is, $\triangleright'$ is obtained from $\triangleright$ by only changing the order of c with its immediate predecessor c'. Then,

1. $\varphi_{\triangleright'}(\mathcal{I}_c) \subseteq \varphi_{\triangleright}(\mathcal{I}_c)$ and
2. $\varphi_{\triangleright'}(\mathcal{I} \setminus \mathcal{I}_c) \supseteq \varphi_{\triangleright}(\mathcal{I} \setminus \mathcal{I}_c)$.

Assuming hard reserves,

- every beneficiary of the preferential treatment category c who is matched by the sequential reserve matching $\varphi_{\triangleright'}$ is also matched under the sequential reserve matching $\varphi_{\triangleright}$, and
- every patient who is not a beneficiary of category c and is matched by the sequential reserve matching $\varphi_{\triangleright}$ is also matched under the sequential reserve matching $\varphi_{\triangleright'}$.

That is, the later a preferential treatment category is processed the more favorable for its beneficiaries and the less favorable for everyone else.

3.4 Competing Interests Under Sequential Reserve Matchings

Theorem 2 motivates a closer look at sequential reserve matchings induced by the following four classes of orders of precedence:

- **Unreserved Last** Π^{ul}: For any precedence $\underline{\triangleright} \in \Pi^{ul}$, each preferential treatment category $c \in \mathcal{C} \setminus \{u\}$ has higher precedence than the unreserved category u.

  Under elements of this class, the unreserved category is processed after all preferential treatment categories. When there is a single preferential treatment category, the resulting sequential reserve matching, first introduced by Hafalir, Yenmez and Yildirim (2013), is uniquely defined.

- **Unreserved First** Π^{uf}: For any precedence $\overline{\triangleright} \in \Pi^{uf}$, each preferential treatment category $c \in \mathcal{C} \setminus \{u\}$ has lower precedence than the unreserved category u.

- **PT-c Optimal** Π^{c}: Fix a preferential treatment category $c \in \mathcal{C} \setminus \{u\}$. For any precedence $\triangleright^{c} \in \Pi^{c}$, the preferential treatment category c has lower precedence than the unreserved category u, which itself has lower precedence than any other preferential treatment category $c' \in \mathcal{C} \setminus \{c, u\}$.

PT-c Pessimal Π_c: Fix a preferential treatment category $c \in \mathcal{C} \setminus \{u\}$. For any precedence $\triangleright_c \in \Pi_c$, the preferential treatment category c has higher precedence than the unreserved category u, which itself has higher precedence than any other preferential treatment category $c' \in \mathcal{C} \setminus \{c, u\}$.

We again obtain our sharpest results for hard-reserves rationing problems.

Theorem 3 Assuming each patient is a beneficiary of at most one preferential treatment category, consider a hard-reserves rationing problem induced by a baseline priority order. Let $\underline{\triangleright} \in \Pi^{ul}$, $\overline{\triangleright} \in \Pi^{uf}$, and $\mu \in \mathcal{M}$ be any matching that is individually rational, non-wasteful and that respects priorities. Then,

$$\varphi_{\overline{\triangleright}}(I_g) \subseteq \mu(I_g) \subseteq \varphi_{\underline{\triangleright}}(I_g).$$

That is, of all matchings that satisfy our three axioms, a sequential reserve matching produces

- the best possible outcome under any unreserved last order of precedence, and
- the worst possible outcome under any unreserved first order of precedence

for general-community patients, in a set inclusion sense.

We conclude our formal analysis with a parallel result for beneficiaries of a given preferred treatment category.

Theorem 4 Assuming each patient is a beneficiary of at most one preferential treatment category, consider a hard-reserves rationing problem induced by a baseline priority order. Fix a preferential tratment category $c \in \mathcal{C} \setminus \{u\}$. Let $\triangleright_c \in \Pi_c$, $\triangleright^c \in \Pi^c$, and $\mu \in \mathcal{M}$

contrast to Theorem 2 (or Theorem 5), which analyzes the impact of changing the processing sequence of an entire category as a block in an environment with multiple preferential treatment categories, Proposition 2 in Dur et al. (2018) analyzes the impact of changing the processing sequence of a single unit in an environment with only one preferential treatment category. Theorems 3 and 4 together can be interpreted as a multiple preferential treatment category generalization of the single preferential treatment category result of Theorem 1 in Pathak, Rees-Jones and Sönmez (2020a). There are also other studies that have examined allocation in the presence constraints such as minimum-guarantee reserves (or lower quotas), upper quotas, and regional quotas. Some of the most related work includes Abdulkadiroğlu (2005), Biro et al. (2010), Kojima (2012), Budish et al. (2013), Hafalir, Yenmez and Yildirim (2013), Westkamp (2013), Ehlers et al. (2014), Echenique and Yenmez (2015), Kamada and Kojima (2015), Kamada and Kojima (2017) Kamada and Kojima (2018), Aygün and Turhan (2016), Aygün and Bo (2016), Bo (2016), Dogan (2016), Kominers and Sönmez (2016), and Fragiadakis and Troyan (2017).

Our paper also introduces the triage protocol design problem into the market design literature. By considering a real-world resource allocation problem, we contribute to the study of formal properties of specific allocation processes in the field and the study of alternative mechanisms. Studies in this vein include those on entry-level labor markets (Roth, 1984; Roth and Peranson, 1999), school choice (Balinski and Sönmez, 1999; Abdulkadiroğlu and Sönmez, 2003; Pathak and Sönmez, 2008, 2013a), spectrum auctions (Milgrom, 2000), kidney exchange (Roth, Sönmez and Ünver, 2004, 2005), internet auctions (Edelman, Ostrovsky and Schwarz, 2007; Varian, 2007), course allocation (Sönmez and Ünver, 2010; Budish, 2011), cadet-branch matching (Sönmez and Switzer, 2013; Sönmez, 2013), assignment of airport arrival slots (Schummer and Vohra, 2013; Schummer and Abizada, 2017), and refugee resettlement (Jones and Teytelboym, 2017; Delacrétaz, Kominers and Teytelboym, 2016; Andersson, 2017).

4 Additional Considerations for Triage Rationing and Future Directions

4.1 Possible Utilization of Excess Units through a Reserve System

While our analysis pertains to the rationing problem of a single entity, say a hospital or a city, it can be extended to multiple entities. This extension would allow for considerations, that can also reduce waste in the system. For example, hospitals in the system can "loan" their unused units to the system, say to a virtual hospital that consists of excess units loaned to the system, and they can earn credit from the system for future use of the units at the virtual hospital when they have a shortage. Hospitals can be incentivized to loan their unused units to the virtual hospital, if their patients receive some priority for some of the units in the virtual hospital. There can be a specific reserve category where priorities may depend on credits earned by the hospitals, while for another category priorities may be determined by clinical criteria only.

priority point systems are expected to comply with these rules. Reserve systems can also comply with the same rules if desired. After all, both a priority point system and a reserve system can be interpreted as accounting systems for applying certain balances of ethical values in managing scarce medical resources.

4.4 Reserve Systems Compared to Constrained Optimization

We brie y contrast a reserve system with an alternative approach that tries to accommodate multiple ethical values using constrained optimization.[20] In a constrained optimization approach, there is an objective function and there are constraints. Perhaps certain ethical values can be aggregated into an objective function, while others can be formulated as constraints. For example, suppose the two ethical goals are to maximize expected health outcomes and non-discrimination. It is a non-trivial task to aggregate these two goals into a single objective function. This is indeed not very di erent than some of the challenges faced in multi-principle point systems. But suppose we are indeed able to nd a representative objective function. Further let us assume that the remaining ethical considerations can be mathematically formulated as constraints. Assuming this maximum can be computed and multiplicities do not cause any issue concerning procedural fairness, we still see several advantages of using a reserve system.

First, a reserve system allows for non-consequentialist ethical goals such as those related to procedural fairness. Clarity on the process by which allocations are determined is an important part of many rationing guidelines. Michigan's standards, for example, state that (Michigan, 2012, page 21)

> procedural justice requires that fair and clear processes be used to make allocation decisions. . .

Furthermore, disability rights groups reject any consideration of probability or length of survival, value. And public acceptance is an essential part of any rationing guideline. For example, New York's guidelines emphasize transparency and state that the \process of developing a clinical ventilator allocation protocol is open to feedback and revision, which helps promote public trust." (Zucker et al., 2015, page 5).

Third, we believe formulating competing objectives within constrained optimization ap-

We hope that the triage rationing protocol we have analyzed will only be necessary in exceptional circumstances during the current pandemic and for future ones. However, even if rationing guidelines are never applied, their mere existence reflects a statement of values.

A Comparative Statics without the Hard-Reserves Assumption

Since the hard-reserves assumption fails to hold in our main application of pandemic rationing, we present in this Appendix a variant of Theorem 2 in the absence of this assumption. There

The bene ciaries of preferential treatment categories c, $c\,$, and $\tilde{c}$ are given as

$$I_c = \{i_1, i_3, i_6\}; \quad I_{c\,} = \{i_2, i_5\}; \quad I_{\tilde{c}} = \{i_4, i_7\};$$

while there are no bene ciaries of preferential treatment categories c^0 and $\hat{c}$: $I_{c^0} = \emptyset$ and $I_{\hat{c}} = \emptyset$. There are also no general-community patients: $I_g = \emptyset$. Suppose π, the baseline priority order of patients, is given as

$$i_1 \succ i_2 \succ i_3 \succ i_4 \succ i_5 \succ i_6 \succ i_7.$$

Also assume that all patients are eligible for all preferential treatment categories besides the unreserved category u.

We consider a sequential reserve matching based on the following order of precedence

$$c^0 \rhd c \rhd c\, \rhd \hat{c} \rhd \tilde{c} \rhd u.$$

This sequential reserve matching matches $\psi_{\rhd}(I) = \{i_1, i_3, i_2, i_4, i_7, i_5\}$ in the order agents are written in this set. In this case,

$$\varphi_{\rhd}(I_c) = \{i_1, i_3\}$$

is the set of matched category-c bene ciaries.

We compare this outcome with the sequential reserve matching under the order of precedence $\rhd^0$ that switches the order of c and c^0, and otherwise, leaves the order of other categories the same as under $\rhd$:

$$c \rhd^0 c^0 \rhd^0 c\, \rhd^0 \hat{c} \rhd^0 \tilde{c} \rhd^0 u.$$

This sequential reserve matching matches $\psi_{\rhd^0}(I) = \{i_1, i_2, i_5, i_3, i_4, i_6\}$ in the order patients are written in this set. In this case,

$$\varphi_{\rhd^0}(I_c) = \{i_1, i_3, i_6\}$$

is the set of matched category-c bene ciaries.

Thus,

$$\varphi_{\rhd^0}(I_c) \supsetneq \varphi_{\rhd}(I_c)$$

although c is is ordered earlier under $\rhd^0$ than under $\rhd$.

Finally the following example shows that in the absence of the hard-reserves assumption, the second conclusion fails even with only two preferential-treatment categories.

Example 2 There is an unreserved category u and two preferential treatment categories c, c^0. There is one medical unit reserved for each category, and the bene ciary groups are [illegible] $= \{i_1, i_2, i_3, i_4\}$, $I_c = \{i_1\}$

.0,298 0 Td [(f)]TJ/F37 10.9091 Tf 5.455 0 Td [(I)]TJ 10.3835F54 10.9701 Tf 4.795 -1.636

Under the sequential reserve matching $_{.}$ the set of patients who are matched is $\{i_1; i_2; i_3\}$, and under the sequential reserve matching $_{.^0}$ the set of patients who are matched is $\{i_1; i_2; i_4\}$. Therefore, when the order of precedence is change from . to $.^0$, a move that is (weakly) detrimental to beneficiaries of category c by Theorem 5, patient i_3, who is not a beneficiary of this category, is made worse off. This shows a change that potentially hurts beneficiaries of one category may hurt other patients as well.

B Proofs

Proof of Theorem 1.

Sufficiency : We first prove that any DA-induced matching is individually rational, non-wasteful and it respects priorities. Let $2 P$ be a preference profile of patients over categories and ; . Suppose $2 M$ is DA-induced from this preference profile.

Individual rationality: Suppose that $(i) = c$ for some $c\ 2\ C$. Then i must apply to c in a step of the DA algorithm, and hence, $c\ _i\ ;$. By construction of $_i$, this means $i\ _c\ ;$. Therefore, matching is individually rational.

Non-wastefulness: Suppose that $i\ _c\ ;$ and $(i) = ;$ for some category $c\ 2\ C$ and patient $i\ 2\ I$. By construction of $_i$, $c\ _i\ ;$ because she is eligible for c. As agent i remains unmatched in , she applies to c in some step of the DA algorithm. However, c rejects i at this or a later step. This means, c should have been holding at least r_c offers from eligible students at this step. From this step on, c always holds r_c offers and eventually all of its units are assigned: $^{1}(c) = r_c$. Hence, matching is non-wasteful.

Respecting priorities: Suppose that $(i) = c$ and $(i^0) = ;$ for two patients $i; i^0\ 2\ I$ and a category $c\ 2\ C$. For this in $^{1}(c)$, who also applied to c in Step 1. Furthermore, since is non-wasteful, $^{1}(c) = r_c$ (as there are unmatched eligible patients for this category, for example j). Therefore, all unmatched patients in are rejected at the rst step of the DA algorithm. Moreover, for every category $c^0 2 C$, all patients in $^{1}(c^0)$ are tentatively accepted by category c^0 at the end of Step 1.

Each unmatched patient in j 2 $^{1}(;)$ continues to apply according to $_j$ to the other categories at which she is eligible. Since respects priorities and is non-wasteful, she is rejected from all categories for which she is eligible one at a time: that is because each of these categories c 2 C continues to tentatively hold patients $^{1}(c)$ from step 1 who have all higher priority than j according to $_c$, as respects priorities. Moreover, by non-wastefulness of , $^{1}(c) = r_c$, as there are unmatched eligible patients (for example j) under .

As a result when the algorithm stops, the outcome is such that, for each category c^0 2 C, all patients in $^{1}(c^0)$ are matched with c^0. Moreover, every patient in $^{1}(;)$ remains unmatched at the end. Therefore, is DA-induced from the constructed patient preferences . ■

Proof of Proposition 1. Let . 2 be a precedence order and ' $_.$ be the associated sequential reserve matching. We show that ' $_.$ is DA-induced from preference pro le $^. =$ ($_i$)$_{i2I}$.

For each patient i 2 I , consider another strict preference relation ^0_i such that all categories that since every patient who is not tentatively accepted by a category $c_1,\ldots,c_{k-1}$ applied to this category in Step k, none of these patients will ever apply to it again; and by the inductive assumption no patient who is tentatively accepted in categories $c_1,\ldots,c_{k-1}$ will ever be rejected, and thus, they will never apply to c_k, either. Thus, the tentative acceptances by c_k will become permanent at the end of the DA algorithm. Thus, this step is identical to Step k of the sequential reserve procedure under precedence order $\rhd$ and the same patients are matched to category c_k in $\varphi_{\rhd}$. This ends the induction.

Therefore, we conclude that $\varphi_{\rhd}$ is DA-induced from patient preference profile $\succ$. ■

Proof of Theorem 2. The sequential reserve matchings $\varphi_{\rhd}$ and $\varphi_{\rhd'}$ match the same patients to the categories with higher precedence than c and c' under both $\rhd$ and $\rhd'$. Let $J \subseteq I$ refer to the set of patients who are available when c' is about to be processed under $\rhd'$ (or equivalently when c is about to be processed under $\rhd$) in the sequential reserve matching procedure.

Two cases are possible: For the unreserved category, either $c' \neq u$ or $c' = u$. We consider these two separately.

Case 1. $c' \neq u$: Then c' is a preferential treatment category as well. Since the problem is hard-reserves, $\varphi_{\rhd}^{-1}(\hat{c}) \subseteq J_{\hat{c}}$ for each $\hat{c} \in \{c, c'\}$ and $\varphi_{\rhd'}^{-1}(\hat{c}) \subseteq J_{\hat{c}}$ for each $\hat{c} \in \{c, c'\}$. Then the order of c' and c do not matter and we have $\varphi_{\rhd}^{-1}(\hat{c}) = \varphi_{\rhd'}^{-1}(\hat{c})$ for each $\hat{c} \in \{c, c'\}$. Hence under both $\rhd$ and $\rhd'$, after c and c' are processed the same set of patients remain unmatched. Since the order of the subsequent categories are the same, both matchings $\varphi_{\rhd'}$ and $\varphi_{\rhd}$ match the same set of patients subsequently to the same categories. Since $\varphi_{\rhd'}$ and $\varphi_{\rhd}$ also match the same patients among $I \setminus J$ to the same categories prior to c and c', we obtain $\varphi_{\rhd'} = \varphi_{\rhd}$, which also implies

$$\varphi_{\rhd'}(I_c) = \varphi_{\rhd}(I_c) \quad \text{and} \quad \varphi_{\rhd'}(I \setminus I_c) = \varphi_{\rhd}(I \setminus I_c);$$

proving the theorem for Case 1.

Case 2. $c' = u$: Then $u \rhd c$ while $c \rhd' u$ and they are consecutively ordered. We are choosing patients among J_c, with respect to the same priority order π to fill either u and c implying that weakly a larger set of category-c beneficiaries are matched to $\{u, c\}$ under $\rhd$ with respect to $\rhd'$:

$$\varphi_{\rhd'}^{-1}(\{c', u\}) \cap J_c \subseteq \varphi_{\rhd}^{-1}(\{c', u\}) \cap J_c. \tag{1}$$

This also implies

$$\varphi_{\rhd'}^{-1}(u) \cap (J \setminus J_c) \supseteq \varphi_{\rhd}^{-1}(u) \cap (J \setminus J_c). \tag{2}$$

Thus, as any general-community category patient in I_g can is only eligible for the unreserved category u because of the hard-reserves feature, by Relationship (2)

$$\varphi_{\rhd'}(I_g) \supseteq \varphi_{\rhd}(I_g).$$

Recall that $\varphi_{\rhd'}$ and $\varphi_{\rhd}$ match the same patients to the categories ordered before c under $\rhd'$ and before u under $\rhd$. There are three cases for the patients in $I \setminus I_g$:

Since category-c beneficiaries are not eligible for any other category ordered after c and u in a hard-reserves problem, then no category-c beneficiary is matched after this step. By relationship (1), we obtain

$$\varphi_{\rhd'}(I_c) \quad \varphi_{\rhd}(I_c):$$

For any preferential treatment category $c' \notin \{c, u\}$ ordered before c under $\rhd'$ and before u under $\rhd$, by relationship (2), we obtain

$$\varphi_{\rhd'}(I_{c'}) \quad \varphi_{\rhd}(I_{c'}):$$

For any preferential treatment category $c' \notin \{c, u\}$ ordered after u under $\rhd'$ and after c under $\rhd$.

As the unreserved category u is processed last in finding $\varphi_{\succ}$ and this is a hard-reserves problem,

$$\varphi_{\succ}^{-1}(u) = \{ i \in I \setminus \cup_{c \in C \setminus \{u\}} \varphi_{\succ}^{-1}(c) : \text{rank}(i; I \setminus \cup_{c \in C \setminus \{u\}} \varphi_{\succ}^{-1}(c); \quad) \quad r_u \}.$$

Take $i \in \quad (I_g)$. Then

a result, $\mathrm{rank}(i; I; \ ) > r_c + r_u$, which is a contradiction to the construction of $\hat{I}_c$ and $\tilde{I}_c$. Therefore, $(i) \neq \emptyset$, which is equivalent to $i \in \ (I_c)$.

We conclude that $\varphi_{\triangleright^c}(I_c) \ \ (I_c)$.

Claim 2: $(I_c) \ \ \varphi_{\triangleright^c}(I_c)$.

Proof. We show that $| \ (I_c)| \ \ |\varphi_{\triangleright^c}(I_c)|$. The claim then follows because both and $\varphi_{\triangleright^c}$ respect priorities. Since both and $\varphi_{\triangleright^c}$ are non-wasteful, the inequality holds if, and only if, the number of category-c beneficiaries assigned to unreserved units in is weakly less than the number of category-c beneficiaries assigned to unreserved units in $\varphi_{\triangleright^c}$ because we are considering a hard-reserves rationing problem.

For every category $c' \in \mathcal{C} \setminus \{u\}$, let

$$\hat{I}_{c'} = \{i \in I_{c'} : \mathrm{rank}(i; I_{c'}; \ ) \ \ r_{c'}\}.$$

This is also the set of patients matched with any preferential treatment category $c' \neq c$ in $\varphi_{\triangleright^c}$ as all preferential treatments other than c are processed before u and c in finding $\varphi_{\triangleright^c}$ and this is a hard-reserves problem.

Then the set of category-c beneficiaries matched to unreserved units in $\varphi_{\triangleright^c}$ is

$$\tilde{I}_c = \{i \in I_c : \mathrm{rank}(i; I \setminus \bigcup_{c' \in \mathcal{C} \setminus \{c,u\}} \hat{I}_{c'}; \ ) \ \ r_u\}$$

as the unreserved category u is processed after all preferential treatment categories other than c and before c in finding $\varphi_{\triangleright^c}$.

Because is DA-induced by Theorem 1, the set of category-c beneficiaries matched to unreserved units in is

$$\{i \in I_c : \mathrm{rank}(i; I \setminus \bigcup_{c' \in \mathcal{C} \setminus \{c,u\}} \ ^{-1}(c'); \ ) \ \ r_u\}.$$

The cardinality of this set is smaller than $|\tilde{I}_c|$ because, by construction, $|\hat{I}_{c'}| \ \ | \ ^{-1}(c')|$ and every patient in $\hat{I}_{c'} \setminus \ ^{-1}(c')$ has a higher priority according to than every patient in $^{-1}(c')$

the rest of the categories will be filled with the same patients, as well, and hence, category-c-beneficiaries who are assigned a unit are identical under both matchings: $\varphi_{\triangleright}(I_c) = \varphi_{\triangleright'}(I_c)$.

Next, suppose that $r_{c'} > |J_{c'}|$. Then $|J_{c'}|$ units of category c' are assigned to category-c' beneficiaries in $\varphi_{\triangleright}$ and $\varphi_{\triangleright'}$ first. The rest of its capacity, which is $r_{c'} - |J_{c'}|$, is filled with respect to priority order π conditional on the eligibility of patients for category c'.

Recall that π_c prioritizes category-c beneficiaries over other patients. Thus, just before we process c' in the sequential reserve matching procedure under $\triangleright'$, the highest priority r_c patients in J_c according to π are no longer available and are matched with c. On the other hand when c' is about to be processed under $\triangleright$, the whole set J_c is available. Under each order of precedence, since the remaining $r_{c'} - |J_{c'}|$ category-c' units are allocated according to the baseline priority order π to all eligible and available patients in $J \setminus J_{c'}$,[23]

$$\varphi_{\triangleright'}^{-1}(\{c', c\}) \cap J_c \subseteq \varphi_{\triangleright}^{-1}(\{c', c\}) \cap J_c; \tag{3}$$

$$\varphi_{\triangleright}^{-1}(c') \setminus \varphi_{\triangleright'}^{-1}(c') \subseteq J_c. \tag{4}$$

To the contrary of what we are trying to prove, suppose there is a patient $i_1 \in \varphi_{\triangleright'}(J_c) \setminus \varphi_{\triangleright}(J_c)$. By Relationship (3), for some category $c_1 \notin \{c, c'\}$, we have $\varphi_{\triangleright'}(i_1) = c_1$ while $\varphi_{\triangleright}(i_1) = \emptyset$. Hence, there are at least 3 categories in $\mathcal{C}$.

Since $\varphi_{\triangleright'}(i_1) = c_1$ and $\varphi_{\triangleright'}$ is individually rational, i_1 is eligible for c_1. Since i_1 is still unmatched after c_1 is processed under $\triangleright$,

$$|\varphi_{\triangleright}^{-1}(c_1)| = r_{c_1};$$

and there exists some patient i_2 such that

$$i_2 \; \pi_{c_2} \; i_1 \quad \text{and} \quad \varphi_{\triangleright}(i_2) = c_1.$$

Moreover, i_2 has to be matched with a category c_2 processed before c_1 under the sequential reserve matching procedure induced by $\triangleright'$, so that i_1 is able to be matched with c_1:

$$c_2 \triangleright' c_1.$$

We also have $c_2 \triangleright c_1$ as $c_1 \notin \{c, c'\}$.

Next, consider patient i_2 who is eligible for c_2 as $\varphi_{\triangleright'}$ is individually rational. Since i_2 is still unmatched after c_2 is processed under $\triangleright$,

$$|\varphi_{\triangleright}^{-1}(c_2)| = r_{c_2};$$

and there exists some patient i_3 such that

$$i_3 \; \pi_{c_2} \; i_2 \quad \text{and} \quad \varphi_{\triangleright}(i_3) = c_2.$$

Moreover, i_3 has to be matched with a category c_3 that is processed before c_2 under sequential reserve matching procedure induced by $\triangleright'$ so that i_2 is matched with c_2:

$$c_3 \triangleright' c_2.$$

This leads to two cases:

[23] For any matching $\mu \in \mathcal{M}$ and set of categories $C' \subseteq \mathcal{C}$, recall that $\mu^{-1}(C') \subseteq I$ is the set of patients matched to the categories in C'.

Case 1. $c_3 \rhd c_2$: Then $c_2 \notin \{c, c'\}$. Since i_3 is still unmatched after c_3 is processed under sequential reserve matching procedure induced by $\rhd$,

$$|\varphi_{\rhd}^{-1}(c_3)| = r_{c_3};$$

and there exists some patient i_4 such that

$$i_4 \pi^{c_3} i_3 \quad \text{and} \quad \varphi_{\rhd}(i_4) = c_3.$$

Moreover, i_4 has to be matched with a category c_4 that is processed before c_3 under the sequential reserve matching procedure induced by $\rhd'$, so that i_3 is able to be matched with c_3:

$$c_4 \rhd' c_3.$$

If $c_4 \rhd c_3$, then $c_3 \notin \{c, c'\}$. Then there are at least five distinct categories ordered according to $\rhd$ as $c' \rhd c \rhd c_3 \rhd c_2 \rhd c_1$; which is a contradiction to $|C| \leq 4$.

Therefore, $c_3 \rhd c_4$, $|C| = 4$, and hence,

$$c_3 = c' \quad \text{and} \quad c_4 = c.$$

Moreover, after c is processed under $\rhd$, $i_1 \in J_c$ remains unmatched. Hence,

$$|\varphi_{\rhd}^{-1}(c)| = r_c.$$

So far we have additionally $|\varphi_{\rhd}^{-1}(c_2)| = r_{c_2}$, $|\varphi_{\rhd}^{-1}(c_1)| = r_{c_1}$, and $|\varphi_{\rhd}^{-1}(c')| = r_{c'}$. Yet i_1 is not matched under $\varphi_{\rhd}$ and is matched under $\varphi_{\rhd'}$. This implies there exists some patient i_0

On the other hand, after c_2 is processed under the sequential reserve matching procedure induced by. , i_2 remains unmatched (because i_2 is matched with c_1, which is processed later since c_2 . c_1), while i is matched with c_2. Then it should be that and matchings ' . and ' .0 are, respectively, as follows:

. :	c^0	c	c	c_1
	i_3 |{z} 2 I c	i_1 |{z} 2 I c		i_2

. 0 :	c	c^0	c	c_1
	i_3 |{z} 2 I c	i_2	j_1 |{z} 2 I c	i_1 |{z} 2 I c

Observe from the above table that i_2 is not matched with c after c is processed the sequential reserve matching procedure induced by, and yet a lower priority patient, j_1 is matched with c under . ^{0}. Then there should be some agent j_2 such that

$$j_2 \ _c \ i_2 \ _c \ j_1 \ _c \ i_1;$$

and

$$' _.(j_2) = c \quad \text{and} \quad ' _{.0}(j_2) \ 2 \ f \ c; c^0 g:$$

If $' _{.0}(j_2) = c$, then j_2 should be matched with c under . instead of j_1, a contradiction. Thus, $' _{.0}(j_2) = c^0$. By Relationship (3), j_2 62J_c.

Now notice that we are exactly at the sam0.303 0s,
d

|{z}

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